The volume

Area of sectoer is a circule
revolving about X−axis $\{r = y = f(x)\}$
$$V = \int_a^b \pi (r)^2 dx = \int_a^b \pi (y)^2 dx$$
revolving about Y−axis $\{r = x = g(y)\}$
$$V = \int_a^b \pi (r)^2 dy = \int_a^b \pi (x)^2 dy$$
revolving about line parallel to X−axis$(y = k)$ $\{r = \lvert k - y \rvert\}$
$$V = \int_a^b \pi (r)^2 dx = \int_a^b \pi (k - y)^2 dx$$
revolving about line parallel to Y−axis$(x = k)$ $\{r = \lvert k - x \rvert\}$
$$V = \int_a^b \pi (r)^2 dy = \int_a^b \pi (k - x)^2 dy$$

Area of is sectoer is a ring
revolving about X−axis$\{r_1 = y_1 = f_1(x), r_2 = y_2 = f_2(x)\}$
$$V = \int_a^b \pi \left[(r_1)^2 - (r_2)^2\right] dx = \int_a^b \pi \left[(y_1)^2 - (y_2)^2\right] dx$$
revolving about Y−axis$\{r_1 = x_1 = g_1(y), r_2 = x_2 = g_2(y)\}$
$$V = \int_a^b \pi \left[(r_1)^2 - (r_2)^2\right] dy = \int_a^b \pi \left[(x_1)^2 - (x_2)^2\right] dy$$
revolving about line parallel to X−axis$(y = k)$ $\{r_i = \lvert k - y_i \rvert\}$
$$V = \int_a^b \pi \left[(r_1)^2 - (r_2)^2\right] dx = \int_a^b \pi \left[(k - y_1)^2 - (k - y_2)^2\right] dx$$
revolving about line parallel to Y−axis$(x = k)$ $\{r_i = \lvert k - x_i \rvert\}$
$$V = \int_a^b \pi \left[(r_1)^2 - (r_2)^2\right] dy = \int_a^b \pi \left[(k - x_1)^2 - (k - x_2)^2\right] dy$$

Examples.

Area of sectoer is a circule
revolving about X−axis (exp2.4)
$\mathbf{R : y = \sqrt{x},\ 0 \le x \le 4}$ $\mathbf{V = \int_0^4 \pi(\sqrt{x})^2 dx = \pi \int_0^4 (x) dx}$
revolving about Y−axis (exp2.5)
$\mathbf{R : y = 4 - x^2, y = 1, x = 0}$ $\mathbf{V = \int_1^4 \pi(\sqrt{4-y})^2 dy = \pi \int_1^4 (4-y) dy}$
revolving about line parallel to X−axis (y = 1)
$\mathbf{R : y = \frac{1}{4}x^2, x = 0, y = 1}$ $\mathbf{V = \int_0^2 \pi(1 - \frac{1}{4}x^2)^2 dx = \pi \int_0^2 (1 - \frac{1}{2}x^2 + \frac{1}{16}x^4) dx}$
revolving about line parallel to Y−axis (x = 1)
$\mathbf{R : y = (x-1)^2, x = 1, y = 1}$ $\mathbf{V = \int_0^1 \pi(1 - \sqrt{y} - 1)^2 dy = \pi \int_0^1 (\sqrt{y})^2 dy}$

Area of sectoer is a ring
revolving about X−axis (exp2.6b)
$R: y = \frac{1}{4}x^2, x = 0, y = 1$ $V = \int_0^2 \pi \left[(1)^2 - (\frac{1}{4}x^2)^2\right] dx$
revolving about Y−axis
$R: y = \sqrt{x}, y = 0, x = 1$ $V = \int_0^1 \pi \left[(1)^2 - (y^2)^2\right] dy = \pi \int_0^1 \left[1 - y^4\right] dy$
revolving about line parallel to X−axis(exp2.7c)
$R: y = \frac{1}{4}x^2, x = 0, y = 1$ (about $y = 2$) $V = \int_0^1 \pi \left[(2 - \frac{1}{4}x^2)^2 - (2-1)^2\right] dx$
revolving about line parallel to Y−axis(exp2.7d)
$R: y = 4 - x^2, y = 0$(about $x = 3$)
$V = \int_0^1 \pi \left[(2 - \frac{1}{4}x^2)^2 - (2-1)^2\right] dx$

Arc length and Area of surface

Arc length	Area of surface(about $x - $axis)
$L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2} dx.$	$S = \int_a^b 2\pi y \sqrt{1 + (\frac{dy}{dx})^2} dx.$
Ex.(1): $y^2 = x^3 \quad 1 \le x \le 4.$	Ex.(1): $y = \sqrt{4 - x^2}$,$-1 \le x \le 1$,
Ex.(2): $y = x^{\frac{2}{3}} - 10, \ 0 \le x \le 8$	Ex.(2): $y = x^3, \quad 0 \le x \le 1$
Exc. (1) $y = \sqrt{1 - x^2}$,$-1 \le x \le 1.$	Exc. :(1) $y = \sqrt{x}, \quad 0 \le x \le 1.$
(2) $y = 4x^{\frac{3}{2}} + 1$, $1 \le x \le 2.$	